THEORY OF FLUIDIZED-BED ELECTRICAL CONDUCTION

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Mechanisms for the passage of electric current through a fluidized bed are discussed, and corresponding estimates for the effective electrical conductivity are given.

Two basic charge-transfer mechanisms in a fluidized bed may be isolated [1]: the first is associated with the passage of current density through the continuous phase of the bed, taking into account the perturbing influence on the potential field of freely suspended particles with a different electrical conductivity, which causes local distortion of the current lines; the second is due to brief random contacts between the randomly pulsating particles of the fluidized bed, leading to the spontaneous formation of conducting circuits of continuously changing configuration. In the case when highly conducting particles are fluidized by a dielectric or poorly conducting medium, the second mechanism may be not only important but even dominating.

When bubbles are present, the bed may be regarded as some hypothetical disperse medium, the continuous phase of which is the dense phase of the bed containing discrete elements of disperse phase (bubbles); the physical parameters of the disperse phase are approximately the same as those of the fluidizing agent.

If particle collision and contact conductivity play a small role, the effective electrical conductivity λ of a homogeneous fluidized bed may be estimated using the theory of [2], developed for heat or mass transfer in disperse media. Specifically, λ may be written in the form

$$\lambda = \lambda_0\beta, \quad \beta = 1 + (\varkappa - 1)\rho\mu, \quad \varkappa = \frac{\lambda_1}{\lambda_0}, \quad \rho = 1 - \varepsilon, \tag{1}$$

the parameter μ being given by the equation

$$\rho\mu J = \rho\left(\frac{4}{3}\pi a^3\right)^{-1}\int_{x=a}^{\infty}\varphi^*(r + x/r)\,n\,dx, \tag{2}$$

where J is the vector of the mean electric-current density in the system; n and φ^* are the vector of the external normal and the electrical potential, averaged over an ensemble of neighboring particles, at the surface of an isolated (sample) particle $x = a$, the center of which is at the point r.

To determine φ^* the following boundary problem must be solved [2]

$$\begin{gathered}\nabla[B(x/a)\nabla\varphi'] = 0, \quad x > a; \quad \Delta\varphi^* = 0; \quad a \geqslant x \geqslant 0,\\ \varphi' \to 0, \quad x \to \infty, \quad \varphi^* < \infty, \quad r = 0,\\ \varphi' - Jx = \varphi^*, \quad \lambda_0 n\nabla\varphi' - \lambda n J = \lambda_1\nabla\varphi^*, \quad x = a,\end{gathered} \tag{3}$$

where

$$\begin{gathered}B(\xi) = 1 - (\varkappa - 1)\rho\mu\frac{27 - 56\xi - 30\xi^2 - \xi^4}{16\xi}, \quad 1 \leqslant \xi < 3,\\ B(\xi) = \beta = 1 - (\varkappa - 1)\rho\mu, \quad \xi \geqslant 3, \quad \xi = \frac{x}{a}.\end{gathered} \tag{4}$$

The substitution $\varphi' = f(x)Jx$ and $\varphi^* = \mu Jx$ [the latter expression identically satisfies Eq. (2)] transforms Eq. (3) to give

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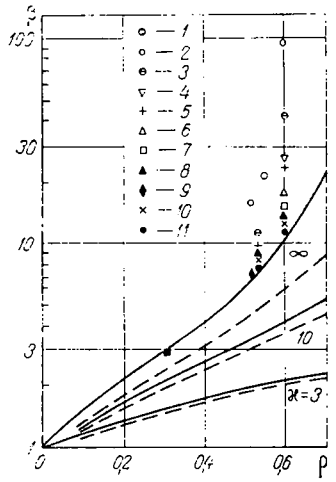

Fig. 1. Dependence of relative diffusional electrical conductivity of the mixture on its concentration for different $\varkappa$ (figures on curve) obtained from the solution of Eq. (5) (continuous lines) and the approximate formula in Eq. (6) (dashed lines). Turner's experimental data: 1) $\varkappa$ = 14,400; 2) 10,100; 3) 2540; 4) 1450; 5) 1040; 6) 580; 7) 400; 8) 290; 9) 220; 10) 190; 11) 160. When ρ = 0.3 all the points combine into one, indicated by the filled square.

$$\xi f'' + 3f' - \frac{d \ln B(\xi)}{d\xi}(\xi f' + f) = 0, \quad \xi = \frac{x}{a},$$
$$f = \mu - 1, \quad f' = \varepsilon(\varkappa - 1)\mu, \quad \xi = 1; \quad f \to 0, \quad \xi \to \infty, \tag{5}$$

where a prime denotes differentiation with respect to ξ. (This is a two-point boundary problem for a second-order ordinary differential equation, and powerful numerical methods are available for its solution. The "redundant" boundary condition in Eq. (5) serves to determine the value of μ, which acts as the eigenvalue of the problem.) The dependence of β on ρ and $\varkappa$ in Eq. (1), determined from the value of μ found numerically from the solution of Eq. (5), is illustrated in Fig. 1.*

If the true function $B(\xi)$ in Eq. (4) is replaced by a step function, equal to unity when $1 \leq \xi < 2$ and to β when $\xi \geq 2$, it is simple to obtain an analytic solution of Eq. (3), use of which in Eqs. (1) and (2) leads to the following approximate formula [2]

$$\beta = [7\varkappa(1-\rho) + 17 + 7\rho]^{-1}\{\varkappa(1 - 11\rho) + 5 - 11\rho + [\varkappa(1 +$$
$$+ 11\rho) + 5 - 11\rho]^2 + [7\varkappa(1-\rho) + 17 + 7\rho][\varkappa(5 + 7\rho) + 7(1-\rho)]^{1/2}\}. \tag{6}$$

The dependence resulting from Eq. (6) is shown by the dashed curves in Fig. 1. It is evident that when $\varkappa \leq$ 1-10 the difference between the accurate and approximate forms of β is insignificant, but at large $\varkappa$ the result obtained on integrating Eq. (5) must be used. For comparison, Fig. 1 also gives the data of Turner (Cambridge) on the electrical conductivity of a homogeneous bed of spherical particles of ion-exchange resin (narrow fractions in the range 0.5-1 mm) fluidized by aqueous medium chloride solutions of different electrical conductivities, obtained in very "pure" conditions.† The agreement between the results of experiment and theory is fair when $\varkappa \approx$ 100-200, and increases steadily with further decrease in $\varkappa$, which, of course, confirms the present theory. However, with decrease in electrical conductivity of the fluidizing solution the

*These calculations were performed by É. Kh. Lipkina, to whom thanks are offered.

†These data were given in the following paper: J. C. R. Turner, "The electrical conductivity of liquid-fluidized beds," A. I. Ch. E. National Meeting, St. Louis (May, 1972). The authors would like to thank Turner for showing them the text of the paper.

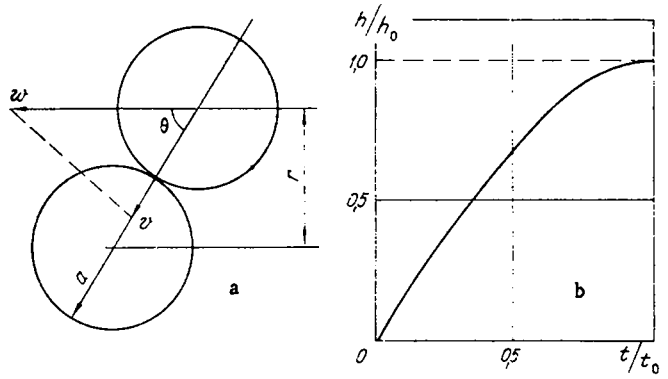

Fig. 2. Collision geometry (a) and dependence of relative distance of compression on the relative collision time (b).

relative effect of contact conduction in a bed of high concentration rises sharply and as a result the effective bed electrical conductivity at $\varkappa \approx 10{,}000$-$15{,}000$ is approximately an order of magnitude higher than at $\varkappa \sim 100$. However, in a bed of relatively low concentration, in which particle collisions are rare or ineffective, contact conduction practically disappears: data for $\varkappa$ in the range 100-1500 appear on the theoretical curve as a single point, corresponding to $\varkappa = \infty$; this point is also shown in Fig. 1. Overall, Fig. 1 provides a good illustration of the role of contact conduction through spontaneously forming particle circuits even in relatively "undisturbed" beds with weak particle pulsations.

Now consider the conductivity of a fluidized bed or its dense phase due to collisions, and the accompanying contacts, between pulsating particles. The continuous phase is assumed to be negligibly small, so that the first charge-transfer mechanism, considered above, may in general be neglected. It is evident that contact conduction depends primarily on the properties of the random pseudoturbulent pulsations of the particles – in particular, on the mean energy of their relative motion – and also on the electrical characteristics of the particle material. Because the factors affecting the intensity of the (in general) anisotropic pseudoturbulent motion of the disperse phase and the appearance of microdischarges in particle collisions are so diverse, there is little hope at present of constructing a rigorous theory of "skeleton" electrical conduction in a fluidization bed which would allow reliable quantitative estimates to be obtained. Therefore, the present analysis is limited to the formulation of some model relations that reflect the main physical properties of this process in qualitative terms. For simplicity, it is assumed that the random pulsations of the particles are isotropic (this is known to be incorrect for beds of very fine particles) and that the current strength is small (so that there are practically no microdischarges and the electrical conductivity is determined by the extent of the area of physical contact formed on collision), and the general method of analysis of skeleton conduction developed in [3] is used.

If an electric current passes through some particle in the direction z, in which its contacts with neighboring particles lie, then by analogy with [3] the potential gradient in the particle, averaged over the particle volume, may be written in the form

$$\langle \nabla\varphi \rangle_z \approx -\frac{j_z}{2\pi a^2 \lambda_1} \ln\frac{1}{\nu}, \quad \nu = \frac{h}{4a} \ll 1, \tag{7}$$

where j_z is the corresponding current strength and h is the distance of compression on contact (denoted by 2δ in [3]). In contrast to the permanent particle contacts in a motionless granular bed considered in [3], in the present case h and ν cannot be assumed to be even approximately the same for all contacts, in view of the considerable energy spread of the colliding particles. In addition, both h and ν change in the course of collision. Therefore the quantity $\ln(1/\nu)$ in Eq. (7) must be averaged both over the collision time and over the ensemble of possible collisions. Assuming that this has been done, the basic discussion of [3] may be repeated, to obtain the following formula for the coefficient of skeleton electrical conductivity of the fluidized bed

$$\Lambda \approx \frac{3}{8}\frac{\pi\zeta\rho}{\langle -\ln\nu \rangle}\lambda_1, \tag{8}$$

where the angle brackets denote the given averaging over the collisions and ζ is a parameter replacing, in the present case, the coordination number of the particles in a fixed granular bed used in [3]. If only binary collisions are considered, ζ is equal to the mean fraction of the time in which any of the particles remains in

contact with its neighbors. Simultaneous collision between three or more particles may evidently only be neglected under the condition that this fraction is small in comparison with unity. If this is not the case, ζ exceeds this fraction and, as the fraction tends to unity, ζ tends to the value of the coordination number characteristic for the type of packing realized in the concentrated fluidized bed. For random packing, this number may be evaluated approximately as the bulk-concentration function of the particles (see [4], for example).

Averaging over the collisions involves successive averaging over the time of a single collision and over the relative particle velocities (or the energy of their relative motion). Below, such a calculation is made on the usual hypotheses of static homogeneity and isotropy of the system, following from the above assumptions. The particles are assumed to be identical ideal spheres and the damping effect of the thin layer of continuous phase between approaching particles on the true collision velocity is disregarded.*

Suppose that the configuration of particles in contact is described by the angle θ between the line joining the particle centers and the direction of the relative velocity w, as shown in Fig. 2. The compression distance will evidently depend on the value $v = w \sin\theta$. On the basis of the considerations in [6], h is described over the duration of collision by the equation

$$m\left(\frac{dh}{dt}\right)^2 + kh^{5/2} = mv^2, \quad k = \frac{8\sqrt{2a}}{15}\cdot\frac{E}{1-\sigma^2}. \tag{9}$$

Hence, in particular, it is simple to obtain [6] an expression for the minimum distance between the particles h_0 and half the collision time t_0†

$$h_0 = \left(\frac{mv^2}{k}\right)^{2/5}, \quad t_0 = \left(\frac{m^2}{k^2 v}\right)^{1/5}\int_0^1 \frac{dx}{\sqrt{1-x^{5/2}}} \approx 1.47\left(\frac{m^2}{k^2 v}\right)^{1/5}. \tag{10}$$

The solution of Eq. (9) under the condition h = 0 at t = 0, using Eq. (10), determines the function $h/h_0 = \Phi(t/t_0)$, given implicitly by the relation

$$\frac{t}{t_0} = \frac{1}{1.47}\int_0^{h/h_0} \frac{dx}{\sqrt{1-x^{5/2}}} \tag{11}$$

and illustrated in Fig. 2b. This function is used to average $\ln(1/\nu) = -\ln\nu$ over the collision time, with the result

$$\langle -\ln\nu \rangle' = -\ln\frac{h_0}{4a} - \int_0^1 \ln\Phi(x)\,dx \approx -\ln\frac{h_0}{4a} + 0.685. \tag{12}$$

In the homogeneous and isotropic situation considered, averaging over the collision geometry in fact means averaging over the impact parameter r (see Fig. 2a). Taking into account that the effective collision cross section is $4\pi a^2$, while $r = 2a\sin\theta$ and $dr = 2a\,d(\sin\theta)$, the result obtained is as follows

$$\langle \ln\sin\theta \rangle' = \frac{1}{4\pi a^2}\int_0^{2a} 2\pi r \ln\sin\theta\,dr = 2\int_0^1 x\ln x\,dx = -0.5, \tag{13}$$

so that after this averaging Eqs. (10) and (12) yields

$$\langle -\ln\nu \rangle'' \approx -\ln\left[\frac{1}{4a}\left(\frac{mw^2}{k}\right)^{2/5}\right] + \frac{4}{5}\langle -\ln\sin\theta \rangle'' - 0.685 \approx -\ln\left[\frac{1}{4a}\left(\frac{mw^2}{k}\right)^{2/5}\right] + 1.085. \tag{14}$$

*This assumption is valid for sufficiently large or heavy particles and for fluidization by gases. If the particles are small and fluidized by liquid drops, most particle interactions are realized through the random pressure and velocity fields of the liquid phase (see the discussion in [5]). This is evidently the cause of the "ineffectiveness" of direct collisions in a bed of low concentration with $\rho = 0.3$ in Turner's experiments (see Fig. 1).

†In [6] central collisions of two spheres were considered. It is readily evident that the presence of a tangential velocity component does not change the energy relation in [6], which leads to Eq. (9), regardless of whether the energy of relative "tangential" motion is conserved or converted to heat.

It only remains to average over the relative velocity of particle collision w. In the homogeneous and isotropic case being considered, the particle distribution over the velocities of random motion is described by the Maxwell formula, in which the mean pulsation energy of a single particle $m\langle W^2\rangle/2$ appears as a parameter. The distribution for the relative velocity is obtained by the standard means after replacing the particle mass m in the Maxwell formula by the reduced mass of the two particles, which for identical spherical particles is m/2. Thus, normalized to unity, the distribution of w takes the form

$$dF(w) = 4\pi\left(\frac{3}{4\pi\langle W^2\rangle}\right)^{3/2}\exp\left(-\frac{3w^2}{4\langle W^2\rangle}\right)w^2dw. \tag{15}$$

Since

$$\langle \ln w\rangle = \int_0^\infty \ln w\,dF(w) = \frac{1}{\sqrt{\pi}}\int_0^\infty\left[\ln\frac{4\langle W^2\rangle}{3} + \ln x\right]e^{-x}\sqrt{x}\,dx \approx \frac{1}{\sqrt{\pi}}\left[\Gamma\left(\frac{3}{2}\right)\ln\frac{4\langle W^2\rangle}{3} + \Gamma'\left(\frac{3}{2}\right)\right] \tag{16}$$

and the values of the gamma function and its derivatives are known, averaging Eq. (14) over the distribution function in Eq. (15) yields the result

$$\langle -\ln\nu\rangle \approx -\ln\left[\frac{1}{4a}\left(\frac{m}{k}\right)^{2/5}\right] - \frac{4}{5}\langle \ln w\rangle + 1.085 \approx -\ln\left[\frac{1}{4a}\left(\frac{m\langle W^2\rangle}{k}\right)^{2/5}\right] + 1.071 \approx \frac{2}{5}\ln T_w + 1.773;$$

$$T_w = \frac{E}{(1-\sigma^2)d_1\langle W^2\rangle}, \tag{17}$$

which depends only on the properties of the particle material and the mean-square pulsation velocity of the particles.

The parameter ζ will now be estimated; in accordance with the foregoing it is identified with the mean fraction of the time spent by a particle in contact with other particles. Evidently, $\zeta = \tau\omega$, where τ is the mean collision time and ω is the collision frequency. Using Eq. (10), averaging over the impact parameter leads, by analogy with Eq. (13), to the relation

$$2\langle t_0\rangle'' \approx 2.94\left(\frac{m^2}{k^2w}\right)^{1/5}\langle \sin^{-1/5}\theta\rangle'' \approx 3.27\left(\frac{m^2}{k^2w}\right)^{1/5}. \tag{18}$$

Averaging this quantity using the distribution function in Eq. (15) leads, by analogy with Eq. (16), to the expression

$$\tau = 2\langle t_0\rangle \approx 3.27\left(\frac{m}{k}\right)^{2/5}\int_0^\infty w^{-1/5}dF(w) = 3.27\frac{2}{\sqrt{\pi}}\Gamma\left(\frac{7}{5}\right)\left(\frac{3m^4}{4k^4\langle W^2\rangle}\right)^{1/10} \approx 3.8T_w^{-2/5}\frac{a}{\langle W^2\rangle^{1/2}}. \tag{19}$$

Evidently, $(1-\tau\omega)\omega_0 = \omega$, where ω_0 is the collision frequency as $\tau \to 0$. Therefore,

$$\zeta = \tau\omega = \frac{\tau\omega_0}{1+\tau\omega_0} \approx \tau\omega_0,\ \tau\omega_0 \ll 1. \tag{20}$$

Enskog theory for a dense gas of solid spheres is used to evaluate ω_0 [7]. Expressing the numerical concentration of particles as the fraction obtained on dividing ρ by the particle volume and replacing 3kT/2 in the usual collision-frequency formula by the mean pulsational energy of a single particle $m\langle W^2\rangle/2$, the following result is obtained

$$\omega_0 = \chi(\rho)\frac{12\rho}{\sqrt{3\pi}}\cdot\frac{\langle W^2\rangle^{1/2}}{a}, \tag{21}$$

where χ is the Enskog factor, describing the change in collision frequency in a dense gas in comparison with an attenuated gas in which the particle size is negligibly small in comparison with the free path length. At small ρ, the Boltzmann–Clausius equation applies

$$\chi(\rho) \approx 1 - 0.625(4\rho) + 0.287(4\rho)^2 + \ldots,\ \rho \ll 1. \tag{22}$$

At values of ρ approaching the concentration of a densely packed bed (i.e., at the onset of fluidization) [7]

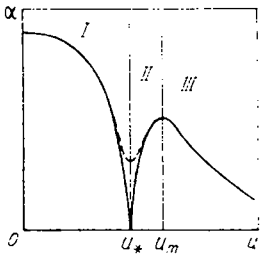

Fig. 3. Characteristic variation in skeleton electrical conductivity of a homogeneous bed with increase in filtration velocity of the liquid phase. Region 1 corresponds to a motionless bed and regions II and III to a fluidized bed: the dashed curve shows the real change in electrical conductivity around the onset of fluidization.

$$\chi(\rho) \approx \frac{(\rho/\rho_*)^{1/3}}{1-(\rho/\rho_*)^{1/3}}, \quad \rho \sim \rho_* - \rho. \tag{23}$$

So as to be specific, Eq. (23) will be used here. Then from Eqs. (19)-(21)

$$\zeta \approx 14.7 \frac{\rho^{4/3}}{\rho_*^{1/3} - \rho^{1/3}} T_w^{-2.5}. \tag{24}$$

This formula is valid if $\tau\omega_0 \ll 1$; it is evidently no longer valid at values of ρ sufficiently close to ρ_*, irrespective of the size T_W. When $\tau\omega_0 \sim 1$, ζ is approximately equal to the coordination number z of a particle in a motionless bed.

Thus, the effective skeleton electrical conductivity in Eq. (8) may be expressed as a function of the bed concentration and also the physical parameters and mean-square pulsational velocity of its particles, and its determination in fact reduces to the gathering of information on this pulsational velocity. The theory of pseudoturbulent motion in finely disperse beds was developed in [5] on the basis of a model according to which the energy input to this motion is due to the work of the fluidizing-agent flux on the random fluctuations of the bed concentration, while particle interaction occurs mainly through the continuous phase. Calculations of various characteristics of such significantly anisotropic pseudoturbulence are given in [8]. Recently, the method of [5] was used to analyze random particle motion in coarsely disperse fluidized beds, when random Magnus forces acting on the particle play a marked role; particle interaction is by direct collision, and it is acceptable to assume a uniform distribution of the pulsational energy over translational and rotational degree of freedom.* The following formula was obtained

$$\langle W^2 \rangle \approx \left(\frac{\rho}{\varepsilon}\right)^2 \left(1 - \frac{\rho}{\rho_*}\right) \Psi(\rho)\, u^2, \tag{25}$$

where u is the filtration velocity of the gas in a homogeneous bed or the dense phase of an inhomogeneous bed, while $\Psi(\rho)$ is a slowly varying function of ρ, depending also in the Reynolds number (Re), and for flow around a single particle also on the parameter. For different ρ and Re this function takes values on the order of $10^{0.5}$-10.

An alternative model of pulsational motion in a very concentrated bed – valid for moving granular beds and possibly at the onset of fluidization, when particle friction is considerable, and particles roll around one another, etc. – was proposed in [9]; this leads to the expression

$$\langle W^2 \rangle \approx \frac{160\rho^{2/3}}{(1-\gamma^2)^2} \left(\frac{d_0}{d_1}\right)^2 (\rho_*^{1/3} - \rho^{1/3}), \tag{26}$$

*These results are reported in the following paper: Yu. A. Buevich, V. V. Butkov, and V. M. Liventsov, Construction of a Rational Theory of a Coarsely Disperse Fluidized Bed [in Russian], Paper Read to the Fifth Scientific-Technical Conference, Ural Polytechnic Institute, Sverdlovsk (February, 1976).

where γ is the velocity-recovery coefficient in collisions between spheres, introduced in [9]. To be specific, Eq. (25) will be used below, but all the discussions may be simply applied to Eq. (26).

In the case when Eq. (24) is valid, simple transformations in Eq. (8) give

$$\alpha = \frac{\Lambda}{\lambda_1} \approx 17{,}3\,\frac{\rho_*^{-2/5}\rho^{47/15}\Psi(\rho)^{2/5}}{\varepsilon^{4/5}T^{2/5}g(T,\rho)}\cdot\frac{(\rho_*-\rho)^{2/5}}{\rho_*^{1/3}-\rho^{1/3}}, \tag{27}$$

which introduces the function

$$g(T,\rho) = \frac{2}{5}\left\{\ln T - \ln(\rho_*-\rho) - \ln\left[\left(\frac{\rho}{\varepsilon}\right)^2\frac{\Psi(\rho)}{\rho_*}\right]\right\} + 1{,}773 \tag{28}$$

and the parameter

$$T = \frac{E}{(1-\sigma^2)\,d_1u^2}, \tag{29}$$

depending only on the properties of the particle and a single process parameter, the filtration velocity u.

When $\tau\omega_0 \sim 1$, it may be assumed that $\zeta = z$ and $\rho = \rho_*$, and Eqs. (8), (17), and (25) give

$$\alpha \approx \frac{3\pi}{8}\cdot\frac{\rho_* z}{g_*(T,\rho_*-\rho)} \approx \frac{1.18\rho_* z}{g_*(T,\rho_*-\rho)}, \tag{30}$$

where

$$g_*(T,\rho_*-\rho) = \frac{2}{5}[\ln T - \ln(\rho_*-\rho)] + I,$$
$$I = -\frac{2}{5}\ln\frac{\rho_*\Psi_*}{\varepsilon_*^2} + 1{,}773, \quad \Psi_* = \Psi(\rho_*). \tag{31}$$

Equation (30) is approximately valid when $\rho_* - \rho < \delta$, where δ may be estimated from the condition $\tau\omega_0 \sim 1$. Suppose that $\rho_* = 0.6$; then

$$\delta \sim (44.1)^{5/3}\rho_*^4(\Psi_*\varepsilon_*^{-2})^{2/3}T^{2/3} \sim 10^3T^{-2/3}. \tag{32}$$

The parameter T is usually very large and δ is small. For example, if $E \sim 10^{11}$ dN/cm^2, $u \sim 10^2$ cm/sec, $d_1 \sim 3$ g/cm^3, and $\sigma \approx 0.3$, then $T \sim 10^{6.5}$ and so $\delta \sim 10^{-1.5} \approx 0.03$. T rises and δ falls with decrease in particle size and density and with increase in Young's modulus of the particle material.

There follows a brief discussion of the dependence of the relative skeleton electrical conductivity on u (or on ρ, which is related to u by the well-known equation expressing the equality of the particle weight to the sum of the Archimedes and hydrodynamic forces) for a homogeneous bed. In the region $u < u_*$ there is a motionless filtered granular bed; in this region, α tends to zero as $u \to u_*$ in accordance with the result in [3], which is shown conventionally by curve I in Fig. 3.† At fluidization numbers close to unity (region II in Fig. 3), α is given by Eq. (30), according to which α increases rapidly in a narrow region of u (or ρ). Finally, in region III, where $\rho_* - \rho > \delta$, Eq. (27) applies, and accordingly α falls fairly rapidly with increase in velocity. At some $u = u_m$ very close to u_*, there is a maximum of the electrical conductivity, which is usually one or two orders of magnitude lower than the electrical conductivity of a motionless granular bed with u = 0. For homogeneous beds, δ is small and $u_m \approx u_*$. This explains why, in experiments with such beds, usually only decrease in electrical conductivity with increase in u is observed (see, e.g., [10] and the data in Fig. 1). The characteristic dependences of α on ρ from Eqs. (27) and (30) for $\rho_* \approx 0.6$, $\Psi(\rho) \approx 5$, and various T are shown in Fig. 4.

In inhomogeneous beds the state of the dense phase is characterized by a value of ρ that differs from ρ_*, about which practically nothing is known. However, it may be assumed that, with increase in fluidization number, ρ decreases from ρ_* to some limiting value independent of the fluidization number. Analogously, the electrical conductivity Λ increases with increase in fluidization number from zero to a certain constant value. The effective electrical conductivity of the bed itself nevertheless varies, as a result of increase in the bulk concentration ρ_B of bubbles in it. Using the model in [2], it is evident that

†In fact, as noted in [3], a proportion of the contacts between particles is retained even at the moment of onset of fluidization. Strictly, therefore, α does not vanish (see the dashed curve in Fig. 3).

$$\Lambda_B = \rho\Lambda, \tag{33}$$

where Λ is the electrical conductivity of the dense phase and β is characterized by the results in Fig. 1 and in Eq. (6), if $\varkappa$ is understood to mean the ratio λ_0/Λ and ρ the value ρ_B. The quantity Λ_B is the statistical mean of the electrical conductivity of a real inhomogeneous bed, and Eq. (33) is valid irrespective of the mechanism of current propagation in the dense phase [i.e., for systems with a highly conducting liquid phase, Λ in Eq. (33) may be replaced by λ]. If $\lambda_0 < \Lambda$, then Λ_B decreases with rise in the fluidization number and, as before, is described by a curve of the same type as in Fig. 1, but this decrease is considerably slower than for a homogeneous bed; the maximum of the electrical conductivity corresponds to fluidization numbers differing considerably from unity. The presence of an electrical-conductivity maximum at the point of transition of the bed to a fluidized state was stated in [11-14], and a maximum was noted in [13, 14]; the fall in electrical conductivity with increase in fluidization number was observed in [10-17] and elsewhere.

The dependence of the coefficient of skeleton electrical conductivity on the characteristics of the particles and the fluidizing agent may easily be derived using the explicit expression for u appearing in the definition of T in Eq. (29).

Thus, electrical conduction in a fluidized bed occurs by one of two mechanisms, distinguished previously in [18]. In the case of poorly conducting particles and a highly conducting fluidizing agent, so that $\lambda = \rho\lambda_0 \gg \Lambda = \alpha\lambda_1$, the first ("diffusional") mechanism operates. In the case when the relation between the conductivities is reversed, the second mechanism, associated with spontaneous particle contacts, comes into force. The changeover between mechanisms occurs at an electrical-conductivity ratio of the phases satisfying the order-of-magnitude relation

$$\varkappa = \frac{\lambda_1}{\lambda_0} \sim \frac{\beta}{\alpha}, \tag{34}$$

this being the condition at which both mechanisms of electrical conduction are important in the bed. A very rough estimate of the electrical conductivity may be obtained by simply summing the coefficients λ and Λ found above. The construction of a more accurate theory within the framework of the method of [2] must involve taking into account the independent transfer of disperse phase over the skeleton, as already noted in [2, 3], and this is a separate problem.

Generally speaking, the conducting circuits responsible for skeleton electrical conductivity are greatly fluctuating. The continuous circuits directly coupling the electrodes which are usual for motionless granular beds may evidently only be formed in fluidized beds at the very onset of fluidization, when $\zeta \approx z$. Outside this range, the charge transfer is by binary collisions between particles which exchange their charges, and continuous circuits, if they appear at all, play no important role, as already noted in [18].

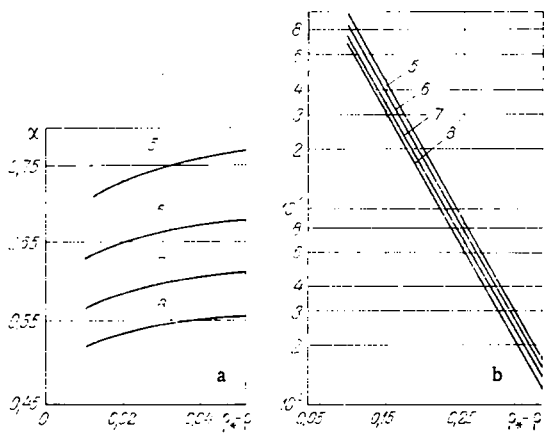

Fig. 4. Dependence of relative skeleton conductivity on $\rho_* - \rho$ for different T according to Eqs. (30) and (27) (a and b, respectively). The values $\rho_* = 0.6$ and $\Psi = 5$ are adopted; the figures on the curves give the values of log T.

In conclusion, note that this calculation of skeleton electrical conduction has entirely disregarded questions relating to ionization of the fluidizing medium and the appearance of microdischarges, so that the application of the results obtained is confined to the passage of relatively weak currents. In addition, if particles of greatly differing potential collide (which is possible if a large current flows in the system), charge transfer may be limited not by the effective contact area but by the capacitance of the particles; attention was drawn to this feature in [18], but it has been disregarded above.

The model developed here has no pretentions at all to being an exhaustive solution of the problem of electrical conduction in fluidized systems, and must be regarded only as some basis for further work.

NOTATION

a	is the particle radius;
d_0 and d_1	are the density of liquid and particle material;
E	is Young's modulus;
F(w)	is the density of the relative particle velocity distribution;
f	is the function defined in Eq. (5);
g and g_*	are the functions introduced in Eqs. (28) and (31);
h	is the distance of compression;
I	is a quantity in Eq. (31);
k	is a parameter in Eq. (9);
m	is the particle mass;
r	is the impact parameter;
T_W and T	are parameters in Eqs. (17) and (29);
t	is the time;
u	is the filtration velocity of fluidizing agent in a homogeneous bed or the dense phase of inhomogeneous bed;
v	is the normal component of relative particle velocity w;
$\langle W^2 \rangle$	is the mean-square pulsational velocity of the particles;
x	is the spatial coordinate;
z	is the coordination number of a particle in the motionless bed;
α	is the relative skeleton electrical conductivity;
B	is a quantity introduced in Eq. (4);
β	is the relative diffusional electrical conductivity;
ν	is the velocity-recovery coefficient in a collision between spheres;
δ	is a quantity defined in Eq. (32);
ε	is the porosity;
ζ	is the effective "coordination number" of a particle in a fluidized bed;
θ	is the angle of collision;
$\varkappa = \lambda_1/\lambda_0$;	
Λ and Λ_B	are the skeleton electrical conductivity and electrical conductivity of an inhomogeneous bed;
λ, λ_0, λ_1	are the diffusional electrical conductivity and electrical conductivity of liquid and solid phases;
μ	is a parameter introduced in Eq. (1);
ν	is the fraction of contact area, equal to $h/4a$;
ξ	is the relative coordinate, x/a;
ρ	is the bulk concentration of particles;
σ	is Poisson's ratio;
τ	is the mean collision time;
Φ	is the function implicitly defined by Eq. (11);
φ	is the electrical potential;
χ	is the Enskog coefficient;
Ψ	is a function introduced in Eq. (25);
ω	is the collision frequency.

Indices

0	is the value at the stage of closest approach of colliding particles;
*	is the value at the onset of fluidization in the bed at collision, as $\tau \to 0$.

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EQUATION OF STATE OF THE LIQUID ALKALI METALS. I.

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Experimental PVT data are given for sodium, rubidium, and cesium in the liquid phase. Procedures and measurement errors are discussed.

Introduction. The liquid alkali metals, in view of the relative simplicity of their structure, can serve as reference items for studying the liquid metal state of a substance. On the other hand, no doubts arise concerning the great prospects for using these metals in various fields of new technology, including their use as coolants.

However, despite the considerable interest which has been shown in recent years toward liquid metals in general, and to the alkali metals in particular [1-3], up to the present time their fundamental characteristics such as the equation of state and its derivatives – compressibility and thermal expansion – remain very little studied.

For the alkali metals in the liquid phase, the temperature range of reliable PVT data is limited, in practice, to 200°C [4-7]. The main quality of data about the thermophysical properties of the fused metals of the first group refers to the line of saturation [8, 9]. In the single-phase region, in addition to the papers mentioned above, PVT data exist on potassium at pressures up to 100 atm and temperatures up to 2050°K [10], also on rubidium in the temperature range 300-1400°C and pressures up to 400 bar [11]. Data exist for cesium on density, obtained by a radioactive method, over a wide range of parameters of state ($P \leq 600$ atm, $600°C \leq T \leq 2500°C$) [12, 13].

The present paper is devoted to an experimental study of the equation of state of liquid cesium, rubidium, and sodium, in the temperature range 500-1000°K and pressures up to 750 atm for cesium and rubidium, and up to 500 atm for sodium.

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